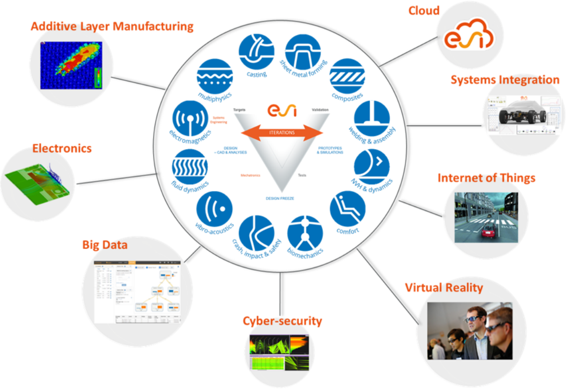
Recovery of differentiation/integration compatibility of meshless operators via local adaptation of the point cloud in the context of nodal integration

Gabriel Fougeron[1,2] Guillaume Pierrot[1] Denis Aubry[2]

[1]ESI Group, Rungis, France

[2]CentraleSupélec, Châtenay-Malabry, France



AN EXPERTISE BASED ON OVER 40 YEARS OF R&D

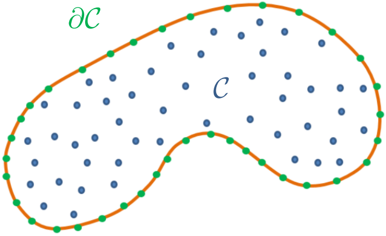
30%

R&D INVESTMENTS / LICENSES REVENUE in FY15

Outline

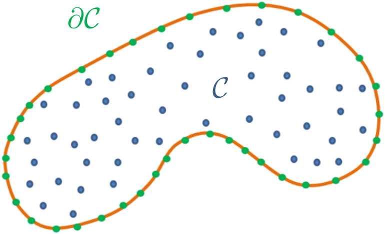
1. Compatibility : what, why ? ... and a new how !
2. Re-interpretation of "background pressure"-like corrections
3. Scaling of the parameter of the method
4. Conclusion and future work

Cloud of points : $\mathcal{C}$
Bondary nodes $\partial\mathcal{C} \subset \mathcal{C}$



Meshless discretization

Cloud of points : $\mathcal{C}$
Bondary nodes $\partial\mathcal{C} \subset \mathcal{C}$



Meshless operators :

- Nodal volume quadrature : $\oint_{\mathcal{C}} f = \sum_{i \in \mathcal{C}} V_i f_i$
- Boundary quadrature : $\oint_{\partial\mathcal{C}} f = \sum_{i \in \partial\mathcal{C}} f_i \mathbf{\Gamma}_i$

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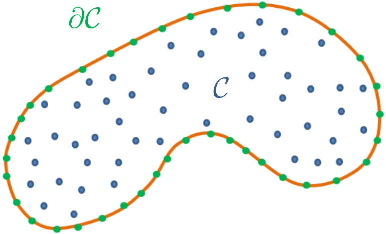

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- Meshless gradient : $V_i \nabla_i f = \sum_{j \in \mathcal{N}(i)} \mathbf{A}_{i,j} f_j$

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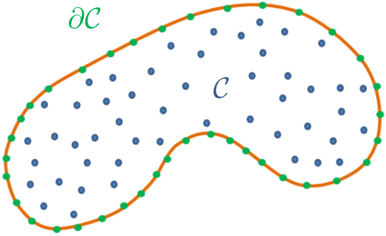

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- Meshless gradient : $V_i \nabla_i f = \sum_{j\in\mathcal{N}(i)} \mathbf{A}_{i,j} f_j$

$$\begin{cases} \nabla_i^{\mathrm{SPH}} f = \sum_j V_j \nabla W(\mathbf{x}_j - \mathbf{x}_i) f_j \\ \nabla_i^{\mathrm{R0}} f = \sum_j V_j \nabla W(\mathbf{x}_j - \mathbf{x}_i)(f_j - f_i) \\ \nabla_i^{\mathrm{R1}} f = \sum_j V_j \mathrm{B}_i \nabla W(\mathbf{x}_j - \mathbf{x}_i)(f_j - f_i) \\ \nabla_i^{\mathrm{MLS}} f = \text{MLS interpolation at } \mathbf{x}_i \end{cases}$$

A dual operator

Integration by parts formula

$$\int_{\Omega} f \nabla \cdot \mathbf{v} + \mathbf{v} \cdot \nabla f \,\mathrm{d}V = \int_{\partial\Omega} f \mathbf{v} \cdot \mathrm{d}\mathbf{S}$$

Discrete counterpart : definition of a dual gradient $\mathbb{\nabla}^*$

$$\int_{\mathcal{C}} f \mathbb{\nabla} \cdot \mathbf{v} + \mathbf{v} \cdot \mathbb{\nabla}^* f = \int_{\partial\mathcal{C}} f \mathbf{v}$$

A dual operator

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$$\oint_{\mathcal{C}} f\mathbb{V}\cdot\mathbf{v} + \mathbf{v}\cdot\mathbb{V}^* f = \oint_{\partial\mathcal{C}} f\mathbf{v}$$

Explicit formula for the dual gradient

$$V_i\mathbb{V}_i^* f = \sum_{j\in\mathcal{N}(i)} (-\mathbf{A}_{j,i} + \delta_{i,j}\mathbf{\Gamma}_i) f_j$$

Plug & play discretization of diffusion equation

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Continuous weak formulation

Find $u \in \mathcal{H}^1(\Omega)$ such that :

$$\begin{cases} \displaystyle\int_\Omega \nabla u \cdot \nabla v = \int_\Omega s v \quad \forall v \in \mathcal{H}^1_0(\Omega) \\ u_{|\partial\Omega} = u_0 \end{cases}$$

Discrete weak formulation

Find $u : \mathcal{C} \to \mathbb{R}$ such that :

$$\begin{cases} \displaystyle\int\!\!\!\!\!\Box_{\mathcal{C}} \nabla u \cdot \nabla v = \int\!\!\!\!\!\Box_{\mathcal{C}} s v \quad \forall v : \mathcal{C} \backslash \partial\mathcal{C} \to \mathbb{R} \\ u_{|\partial\mathcal{C}} = u_0 \end{cases}$$

Linear patch test

Equivalent nodewise formulation

Find $u : \mathcal{C} \to \mathbb{R}$ such that :

$$\begin{cases} -\mathbb{V}_i^* \cdot \mathbb{V}u = s_i & \forall i \in \mathcal{C} \backslash \partial\mathcal{C} \\ u_{|\partial\mathcal{C}} = u_0 & \end{cases}$$

Linear patch test

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Necessary conditions for the linear patch test

- $\nabla x = \mathrm{I}_d$
- $\nabla^* 1 = 0$

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Necessary conditions for the linear patch test

- $\mathbb{V}x = \mathrm{I}_d$
- $\mathbb{V}^* 1 = 0$ Compatibility !

Linear patch test

Equivalent nodewise formulation

Find $u : \mathcal{C} \to \mathbb{R}$ such that :

$$\begin{cases} -\mathbb{V}_i^* \cdot \mathbb{V} u = s_i & \forall i \in \mathcal{C} \backslash \partial \mathcal{C} \\ u_{|\partial \mathcal{C}} = u_0 & \end{cases}$$

Necessary conditions for the linear patch test

- $\mathbb{V} x = \mathrm{I}_d$
- $\mathbb{V}^* 1 = 0$

$\Leftrightarrow$ Discrete Stokes formula : $\fint_{\mathcal{C}} \mathbb{V} u = \fint_{\partial \mathcal{C}} u$

Compatibility recovery : correction

Corrected gradient

$$\nabla_i^c u = \nabla_i u + \sum_{j \in \mathcal{N}(i)} \boldsymbol{\mu}_{i,j} (u_j - u_i - \nabla_i u \cdot (\mathbf{x}_j - \mathbf{x}_i))$$

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Correction preserves linear consistency :

$$\forall \boldsymbol{\mu}_{i,j}, \; \mathbb{V}\mathbf{x} = \mathrm{I}_d \Rightarrow \mathbb{V}^c \mathbf{x} = \mathrm{I}_d$$

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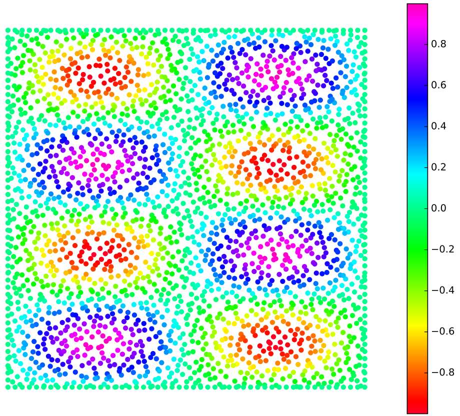
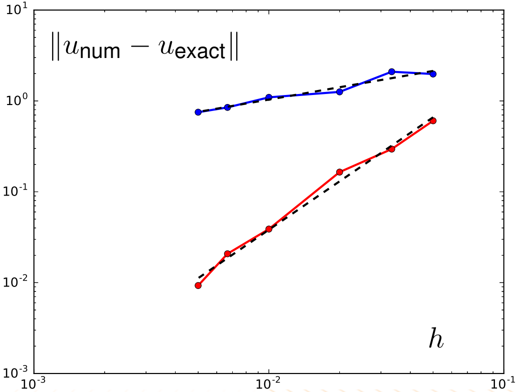
Correction equations

Solve $\mathbb{V}^{c*}1 = 0$ for $\boldsymbol{\mu}_{i,j}$ given $\mathbb{V}$



Halton sequences

$$
\begin{aligned}
s &= 20\pi^2 \sin(2\pi x)\sin(4\pi y) \\
u &= \sin(2\pi x)\sin(4\pi y)
\end{aligned}
$$



Non-corrected

$\|u_{\text{num}} - u_{\text{exact}}\| \propto h^{0.45}$

Corrected

$\|u_{\text{num}} - u_{\text{exact}}\| \propto h^{1.77}$

$$\mathbb{\nabla}_i^{\text{R1}} f = -\sum_{j \in \mathcal{C}} V_j \mathbb{B}_i \nabla W_h(\mathbf{x}_j - \mathbf{x}_i)(f_j - f_i)$$

$$\mathbb{B}_i^{-1} = -\sum_{j \in \mathcal{C}} V_j \nabla W_h(\mathbf{x}_j - \mathbf{x}_i)(\mathbf{x}_j - \mathbf{x}_i)^T$$

☺ Pros

- We recover almost second order convergence

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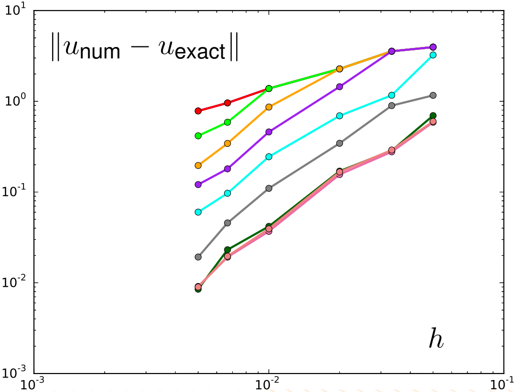
- We recover almost second order convergence

☹ Cons

- An additional global linear system to solve
  ... every timestep !
- No explicit formula for the gradient



$\|\mathbb{\nabla}^*1\| \leq 50$

$\|\mathbb{\nabla}^*1\| \leq 20$

$\|\mathbb{\nabla}^*1\| \leq 10$

$\|\mathbb{\nabla}^*1\| \leq 5$

$\|\mathbb{\nabla}^*1\| \leq 2$

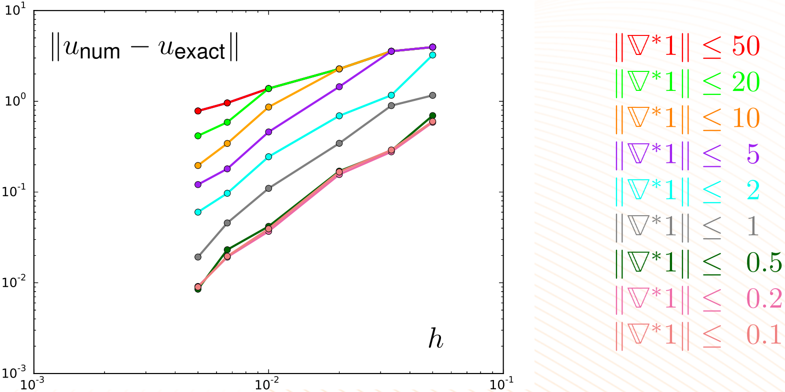
$\|\mathbb{\nabla}^*1\| \leq 1$

$\|\mathbb{\nabla}^*1\| \leq 0.5$

$\|\mathbb{\nabla}^*1\| \leq 0.2$

$\|\mathbb{\nabla}^*1\| \leq 0.1$

Approximate compatibility

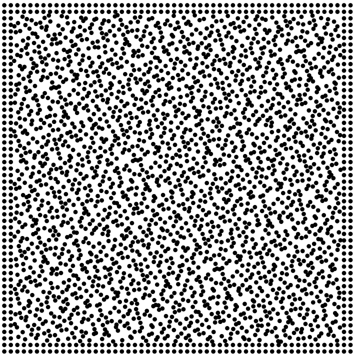


Observation : Keep $\|\nabla^* 1\| = \mathcal{O}(1)$ instead of the usual $\|\nabla^* 1\| = \mathcal{O}(h^{-1})$

$\Rightarrow$ recover almost second order convergence !

Another idea :

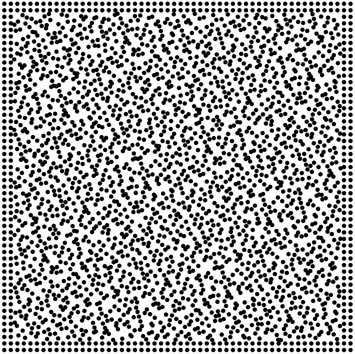
Go from :



$$\|\nabla^{R1*}1\| \approx 36.3$$

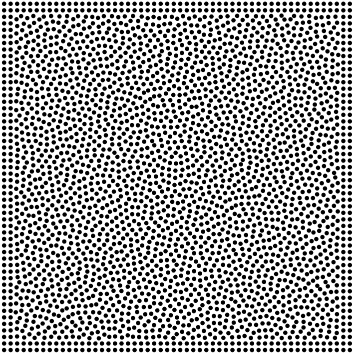
Another idea :

Go from :



$\|\mathbb{\nabla}^{R1*}1\| \approx 36.3$

To :



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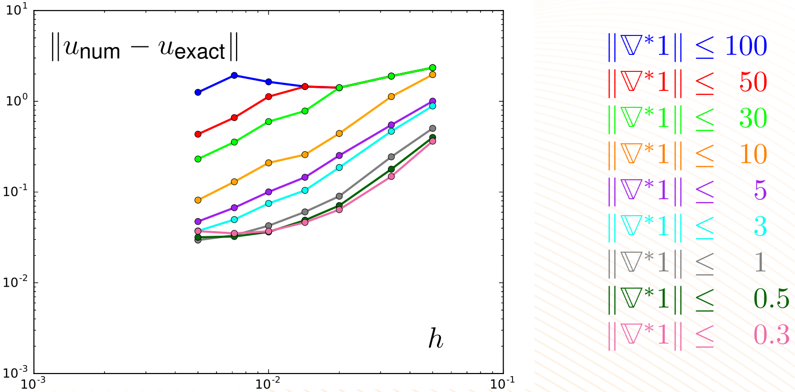
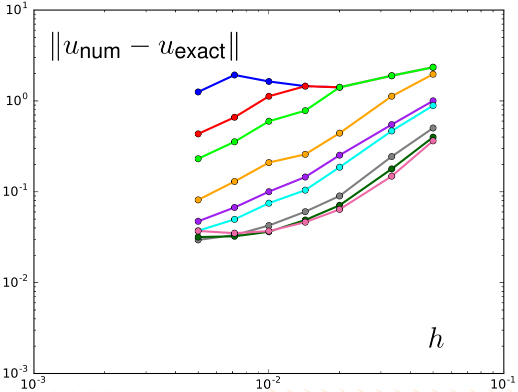


$$\mathbb{V}_i^{\mathrm{R0}} f = -\sum_{j \in \mathcal{C}} V_j \nabla W_h(\mathbf{x}_j - \mathbf{x}_i)(f_j - f_i)$$



$\|\mathbb{\nabla}^* 1\| \leq 100$

$\|\mathbb{\nabla}^* 1\| \leq 50$

$\|\mathbb{\nabla}^* 1\| \leq 30$

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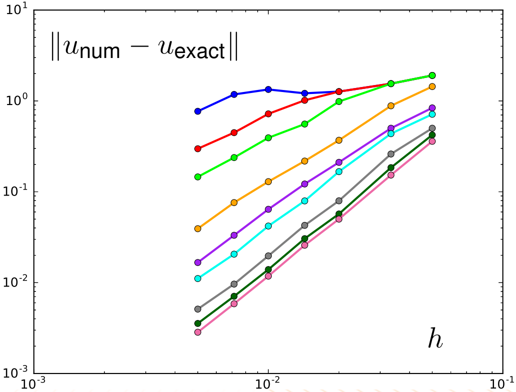
$$\mathbb{\nabla}_i^{\mathrm{R0}} f = -\sum_{j \in \mathcal{C}} V_j \nabla W_h(\mathbf{x}_j - \mathbf{x}_i)(f_j - f_i)$$

$$\mathbb{\nabla}_i^{\mathrm{R0}} \mathbf{x} \neq \mathrm{I}_d$$



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What does it mean ?

Statics $\Rightarrow$ A gradient-specific "remeshing" procedure

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Statics ⇒ A gradient-specific "remeshing" procedure

Dynamics ⇒ A gradient-specific ALE source term

Playing with node coordinates

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Statics $\Rightarrow$ A gradient-specific "remeshing" procedure

Dynamics $\Rightarrow$ A gradient-specific ALE source term

The idea is not new !

- XSPH (Monaghan)
- Background pressure
- Transport-velocity formulation [Adami 2013]
- Fickian-based shifting [Lind 2011]

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Background pressure and equivalents

$$\begin{cases} m_i \dfrac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = \mathbf{f}_i \\ \dfrac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}t} = \mathbf{v}_i \end{cases}$$

Background pressure and equivalents

$$\begin{cases} m_i \dfrac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = \mathbf{f}_i \quad {\color{red} - V_i \nabla_i^* P_u} \\ \dfrac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}t} = \mathbf{v}_i \end{cases}$$

Background pressure and equivalents

$$\begin{cases} m_i \dfrac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = \mathbf{f}_i \quad {\color{red}-V_i P_u \mathbb{V}_i^* 1} \\ \dfrac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}t} = \mathbf{v}_i \end{cases}$$

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The simplest example

Classical and renormalized SPH of order 0

$$V_i \mathbb{\nabla}_i^{SPH*} 1 = \frac{1}{2} V_i \mathbb{\nabla}_i^{R0*} 1 = V_i \mathbb{\nabla}_i^{SPH} 1 = - \sum_{j \in \mathcal{N}(i)} V_i V_j \nabla W(\mathbf{x}_j - \mathbf{x}_i)$$

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Constant quadrature : V_i independent of $(\mathbf{x}_j)_{j \in \mathcal{C}}$

$$V_i \mathbb{V}_i^{SPH*} 1 = \frac{\partial}{\partial \mathbf{x}_i} V_{tot}$$

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Constant quadrature : V_i independent of $(\mathbf{x}_j)_{j \in \mathcal{C}}$

$$V_i \mathbb{\nabla}_i^{SPH*} 1 = \frac{\partial}{\partial \mathbf{x}_i} V_{tot}$$

$$V_{tot} = \sum_k V_k \sum_j V_j W(\mathbf{x}_j - \mathbf{x}_k)$$

$$= \oint_{\mathcal{C}} < 1 >$$

Density-based nodal quadrature

$$V_i = \frac{1}{\sum_j W(\mathbf{x}_j - \mathbf{x}_i)}$$

$$V_i \mathbb{\nabla}_i^{Adami*} 1 = - \sum_{j \in \mathcal{N}(i)} (V_i^2 + V_j^2) \nabla W(\mathbf{x}_j - \mathbf{x}_i)$$ [Adami 2013]

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$$= \frac{\partial}{\partial \mathbf{x}_i} V_{tot}$$

$$V_{tot} = \sum_k V_k = \oint_{\mathcal{C}} 1$$

ALE source term

$$\frac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}t} = -\gamma V_i \mathbb{\nabla}^* 1$$

$$\mathcal{L}((\mathbf{x}_i)_{i\in\mathcal{C}}) = -V_{tot}$$

ALE source term

$$\frac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}t} = -\gamma V_i \mathbb{V}^* 1$$

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$\Rightarrow$ Convergence towards a node distribution where $\frac{\partial \mathcal{L}}{\partial \mathbf{x}_i} = V_i \mathbb{V}_i^* 1 = 0$

ALE source term

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$$\mathcal{L}((\mathbf{x}_i)_{i \in \mathcal{C}}) = -V_{tot}$$

Background pressure

$$\begin{cases} \dfrac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = -\alpha \mathbf{v}_i - \gamma V_i \nabla^* 1 \\ \dfrac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}t} = \mathbf{v}_i \end{cases}$$

$$\mathcal{L}((\mathbf{x}_i, \mathbf{v}_i)_{i \in \mathcal{C}}) = \frac{1}{2} \sum_i \|\mathbf{v}_i\|^2 - \gamma V_{tot}$$

$\Rightarrow$ Convergence towards a node distribution where $\dfrac{\partial \mathcal{L}}{\partial \mathbf{x}_i} = V_i \nabla_i^* 1 = 0$

The simplest dynamic problem : advection

$$\frac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}t} = \mathbf{v}(\mathbf{x}_i) \quad \forall i \in \mathcal{C} \backslash \partial \mathcal{C}$$

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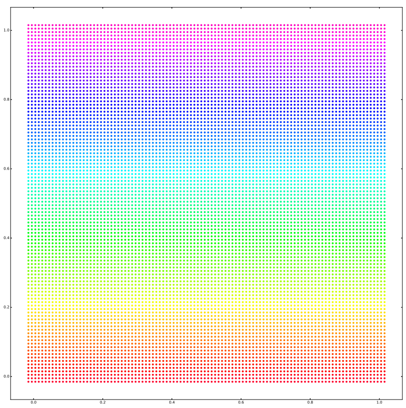
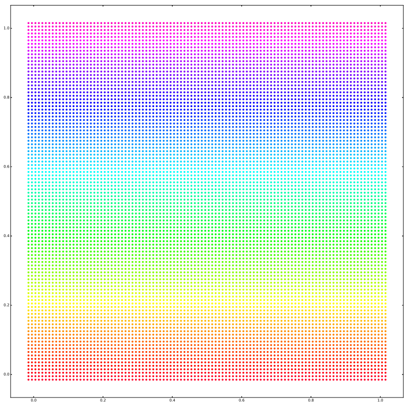
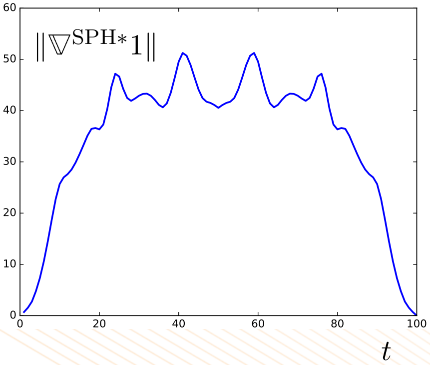


Taylor-Green vortices

$$\frac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}t} = \mathbf{v}(\mathbf{x}_i) \quad \forall i \in \mathcal{C} \backslash \partial\mathcal{C}$$





Approximate advection

$$\begin{cases} \dfrac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}t} = \mathbf{v}(\mathbf{x}_i) - \mathbf{c}_i \quad \forall i \in \mathcal{C}\backslash\partial\mathcal{C} \\ \mathbf{c}_i = \gamma \mathbb{V}^{*\mathrm{SPH}} 1 \end{cases}$$

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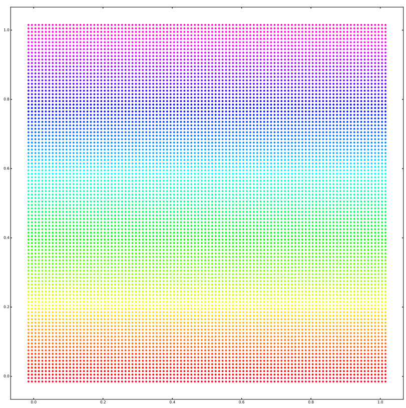
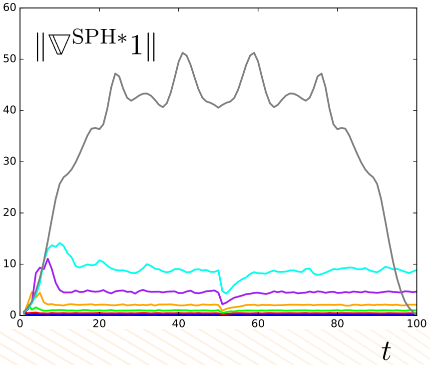
The ALE speed

Approximate advection

$$
\begin{cases}
\dfrac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}t} = \mathbf{v}(\mathbf{x}_i) - \mathbf{c}_i & \forall i \in \mathcal{C} \backslash \partial\mathcal{C} \\
\mathbf{c}_i = \gamma \mathbb{V}^{*\mathrm{SPH}} 1
\end{cases}
$$





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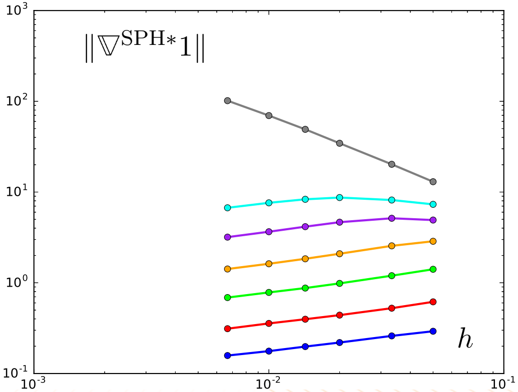


$\gamma = 0.3$

$\gamma = 0.1$

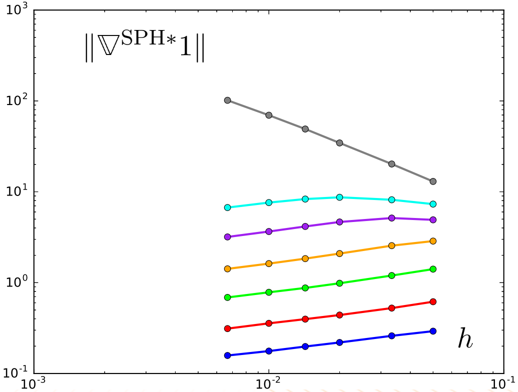
$\gamma = 0.03$

$\gamma = 0.01$

$\gamma = 0.003$

$\gamma = 0.001$

$\gamma = 0$



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$\gamma = 0.01$

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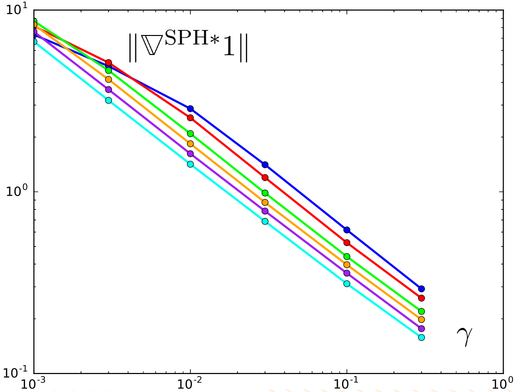
$\gamma = 0.001$

$\gamma = 0$

$$\|\mathbb{V}^{SPH*}1\| \propto h^{0.35}$$
$$\|\mathbf{c}\| \propto h^{0.35}$$

Influence of parameter γ



$h = 3.3 \cdot 10^{-2}$
$h = 2 \cdot 10^{-2}$
$h = 1.4 \cdot 10^{-2}$
$h = 1 \cdot 10^{-2}$
$h = 6.6 \cdot 10^{-3}$

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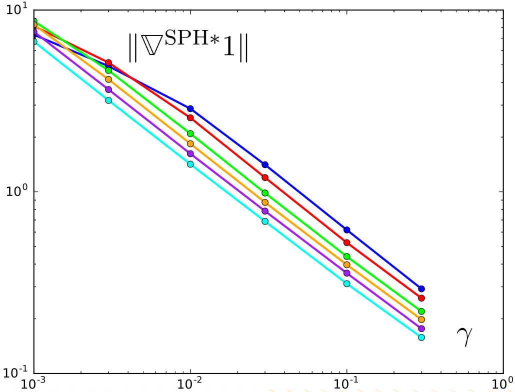
$$\|\mathbf{c}\| \propto h^{0.35}$$



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$$\|\mathbb{V}^{SPH*}1\| \propto h^{0.35}\gamma^{-0.65}$$
$$\|\mathbf{c}\| \propto h^{0.35}\gamma^{0.35}$$

$$\|\mathbb{\nabla}^{SPH*}1\| \propto h^{0.35}\gamma^{-0.65}$$
$$\|\mathbf{c}\| \propto h^{0.35}\gamma^{0.35}$$

$$\gamma = \mathcal{O}(\sqrt{h}) \;\Rightarrow\; \begin{cases} \|\mathbb{\nabla}^{SPH*}1\| = \mathcal{O}(1) \\ \|\mathbf{c}\| = \mathcal{O}(\sqrt{h}) \end{cases}$$

Fick-based diffusion [Lind 2011]

$$\begin{cases} \dfrac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}t} = \mathbf{v}(\mathbf{x}_i) - \mathbf{c}_i & \forall i \in \mathcal{C}\backslash\partial\mathcal{C} \\ \mathbf{c}_i = \gamma \mathbb{V}^{*\mathrm{SPH}} 1 \end{cases}$$

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... then uses $\mathbb{V}^{\mathrm{R1}}$ for Poisson equation !

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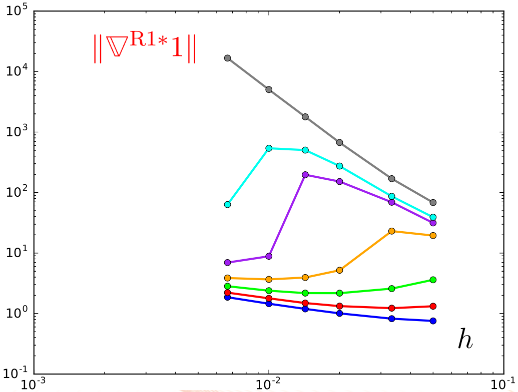
$$\begin{cases} \frac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}t} = \mathbf{v}(\mathbf{x}_i) - \mathbf{c}_i \quad \forall i \in \mathcal{C} \backslash \partial \mathcal{C} \\ \mathbf{c}_i = \gamma \mathbb{V}^{*\mathrm{SPH}} 1 \end{cases}$$



$\gamma = 0.3$
$\gamma = 0.1$
$\gamma = 0.03$
$\gamma = 0.01$
$\gamma = 0.003$
$\gamma = 0.001$
$\gamma = 0$

Summary and future work

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- Re-interpretation of several background-pressure like procedures within an operator framework
- Proved convergence of the method to compatible steady state for several existing gradient
- We proposed a proper scaling for the parameter of the method

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Ongoing and future work

- Clarify the role of volume related functionals
- Extend methodology to handle other boundary treatments

Thanks for your attention !

gabriel.fougeron@esi-group.com guillaume.pierrot@esi-group.com

denis.aubry@ecp.fr